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LETTER TO THE EDITOR

Traffic models with disorder

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Abstract. We propose traffic models with quenched noise in order to investigate the effects of disorder on the flow properties in complex transport systems. In the one-dimensional model we studied in detail, we observed instabilities with respect to disorder and transitions between various modes of propagation.

In recent years much attention has been attracted by transport phenomena in complex systems, including such special examples as flows in granular materials [1, 2], data propagation in complex computer networks [3], collective migration in biological systems [4] and highway traffic flow [5]. A common aspect of these examples is the existence of jamming (or jamming transition): if there are too many particles in the system they inhibit each other's motion and this results in halting of the flow in extensive regions. This effect determines the non-trivial transport properties of such flows, and it is likely to exist in most of the driven or self-driven systems containing moving particles with dissipative interaction.

To understand the mechanism leading to jamming in realistic traffic situations several models have been introduced. For the 1D case a cellular automaton model has been investigated both numerically [5, 6] and theoretically [7]. There is also a continuum equation approach for this case [8] and pattern formation aspects based on the flow characteristics have also been studied [9]. For the 2D case, which is the more realistic one, cellular automaton models have been studied [10–12], and signs of first-order phase transitions have been found.

We present a model for one-dimensional flow in the presence of quenched noise. Our model is based on the 1D model of Schreckenberg and Nagel [5], which has been thoroughly analysed by Nagel and Herrmann [6].

N cars (particles or data packets) are arranged in a one-dimensional lattice (path) of length L with periodic boundary conditions. The position of the cars are denoted by x_i ($i = 0, 1, \ldots, L-1$) and to each of them a velocity (v_i) is assigned which has integer values and can vary in the range $0 \leqslant v_i \leqslant 5$, corresponding to the choice $v_{\max} = 5$ in the original model. We assign to each position along the path an integer s_i corresponding to the 'inverse permeability' of that part of the path: the rougher the road the larger is the value of s_i. The s_i values are fixed in time, i.e., they represent a quenched disorder.

At each time step the position and the velocity of each car are updated simultaneously (parallel update) according to the following rules:

Step 1. If $v_i < 5$ and the distance from the car ahead ($d = (x_{i+1} - x_i) \bmod L$) is greater than $v_i + 1$ then v_i is increased by one ($v_i \leftarrow v_i + 1$, acceleration);

Step 2. If the distance from the car ahead is less than $v_i + 1$ then $v_i \leftarrow d - 1$ (slowing down);

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Step 3. If $v_i > 0$ then v_i is randomly decreased by 1 with probability p (random slowing down);

Step 4. The car is moved forward: variable y is set to x_i and then y is incremented by 1 while the condition

$$\sum_{j=x_i}^{y} s_j < v_i$$

is satisfied.

Step 5. The velocity is set to the actual forward move of the car:

$$v_i \leftarrow (y - x_i) \bmod L$$

and its position is updated: $x_i \leftarrow y$.

In the case of $s_i \equiv 1$ this update corresponds to a forward move by v_i sites as in the model in [5]. The sites with $s_i > 1$ will cause slowing down of the flow, since they make the cars move a shorter distance than they would travel in the case of free propagation ($s_i \equiv 1$). For example a car entering a region with $s_i \equiv 2$ at speed $v = 4$ moves only two sites and its speed decreases to $v = 2$. Therefore sites with $s_i > 1$ are referred to as *curbs* in the road. Allowing the s_i values to change in time would cause the same random slowing down effect as step 3 does, which is equivalent to a shot noise. Throughout this letter we will focus on the case when the $s_i - s$ can have values 1 and 2 only. The density of $s_i = 2$ sites (curbs) is another parameter in the model

$$r = \frac{\#\text{ of curbs}}{\text{system size}}.$$

Let us now introduce the quantities used for characterizing the flow. The number of cars (N) is preserved in the present dynamics, so it is straightforward to use the *global* density

$$\rho = \frac{N}{L}$$

as a control parameter. One can introduce the local density of cars at site x which is defined as

$$\rho(x) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} n_x(t)$$

where $n_x(t) = 0(1)$ if site x is empty (occupied) at time step t. The global density can be expressed through the local density in the following way:

$$\rho = \frac{1}{L} \sum_{x=0}^{L-1} \rho(x).$$

The average car flow is defined by

$$j = \lim_{T\to\infty} \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} n_{x,x+1}(t)$$

where $n_{x,x+1}(t) = 1$ if a car passed the border between sites x and $x+1$ at time step t, and zero otherwise. Note that j is independent of x, this is due to the fact that in the stationary state the flow should be uniform at each site due to car conservation.

The value of t_0 is irrelevant in the infinite time limit ($T \to \infty$). In the simulations we used a large, but finite averaging time T, and also a large enough time t_0 for relaxation. The plots in the figures were obtained by averaging over $20 \times L$ timesteps (full updates) after relaxation taking $5 \times L$ timesteps. The stationarity was checked by comparing averages over different times.

Using the quantities defined above we have performed runs for several different parameter values in order to find the change in the fundamental diagram, i.e., in the flow (j) versus global density (ρ) plot.

Let us first consider the deterministic case ($p = 0$ in step 3). Figure 1 shows the j–ρ curve at different amounts of curbs present in the system of size $L = 1024$. For the case when only *one* curb is present the plot significantly differs from the curb free ($r = 0$) case. It has an overshoot unlike the curves obtained by the original model. This curve was obtained by averaging over the quenched randomness, for a given realization of the quenched noise the overshoot is more characteristic. The simulations for a larger system ($L = 2048$) gave the same plot so we can draw the conclusion that the original model is *unstable* against the quenched noise. We conjecture that this is the case even in the thermodynamic limit $L \to \infty$. The one-curb case can be interpreted as a change in the boundary condition. An analogous situation has been intensively studied for the related asymmetric exclusion model [13, 14].

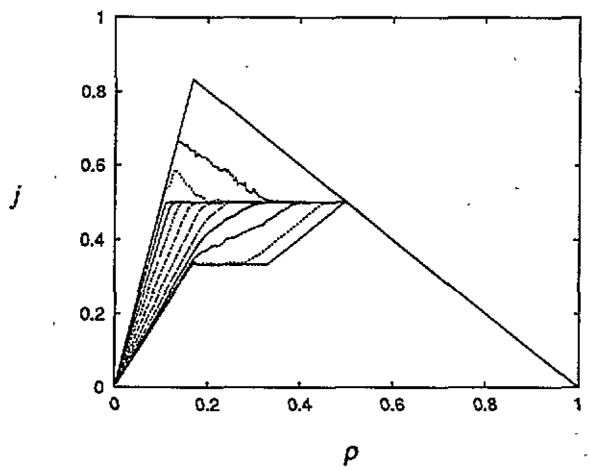

Figure 1. The fundamental diagram of the deterministic variant of the model for various densities of the curbs in a system of size $L = 1024$. The top curve is for the curb-free ($r = 0$) case. The next curve downwards is for a single curb ('$r = 1/L$'), and the further curves are for increasing (finite) curb densities ($r = 0.01, 0.1, 0.2 \ldots 1$).

As we add more curbs to the system the fundamental diagram first changes to a shape with a plateau at $j = 0.5$ then continuously crosses over to a stepwise linear curve with a plateau at $j = 0.333$. Note that for high car densities the system is insensitive to the quenched disorder since, even for $s_i \equiv 1$, jamming is present.

When adding a small shot noise to the system ($p = 0.01$) the flow properties are changed again. Figure 2 shows the fundamental diagram for this case. The curve for one curb is smoothed out by the noise, although a very small overshoot is still present. The stepwise linear curve for $r = 1$ loses stability and is replaced by the $j/\rho = 1$ curve for $\rho < 0.5$.

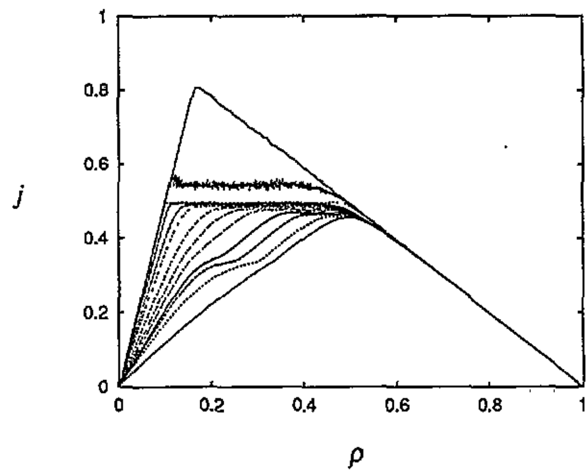

Figure 2. The same diagram as figure 1 but for the model with very small shot noise ($p = 0.01$).

Next we give a theoretical argument for the observed instability at high curb densities ($r = 1$). When the global density of cars is small the interaction represented in the model by step 2 can be neglected. For this case one can write down a master equation for the velocity distribution of a single car. Let us introduce the probability p_v^t of a car having speed v at time t. After a single update of the system this car is assigned the speed w with certain probability $M_{w,v}$. We have from $p_w^{t+1} = \sum_{v=0}^{5} M_{w,v} p_v^t$ the following equation for the stationary speed distribution:

$$p_w = \sum_{v=0}^{5} M_{w,v} p_v$$

which is an eigenvector problem for M. The elements of the matrix M are exactly calculated by numerically averaging over all possible realizations of the relevant part of the road ahead of a car. Since v_{max} is finite (5) only a finite number of sites need to be considered. With this method both random slowing down and quenched curbs can be exactly considered, and for M an analytic expression is found. The only approximation is that we neglect the interaction between the cars, which can be handled by introducing the combined speed–distance distribution.

The results for the average speed ($\bar{v} = j/\rho = \sum_{v=0}^{5} v p_v$) obtained by this method are presented in figure 3. These plots clearly indicate that for $r = 1$ the $j/\rho = 2$ solution becomes unstable for $p > 0$ and replaced by $j/\rho = 1$, as we have seen in the previous section. For $r = 0$ the calculation reproduces the trivial result

$$j/\rho = 5 - p.$$

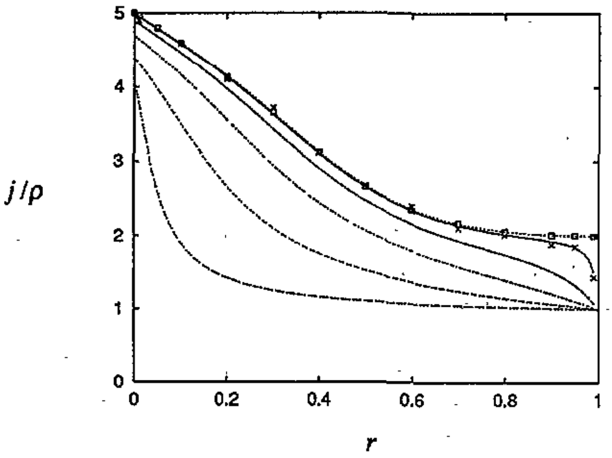

Figure 3. The average speed (j/ρ) in the low-car-density limit. The curves show the results of the analytic calculation for no shot noise (top curve) and for increasing shot noise (from top to bottom: $p = 0.01, 0.1, 0.3, 0.6, 0.9$). The points represent the average speed obtained from the curves on figure 1 (boxes, $p = 0$) and figure 2 (crosses, $p = 0.01$).

Figure 4. Motion of the cars in the spacetime domain. Each dot represents a car, they are moving downwards in the figure, subsequent iterations are plotted side-by-side, so the horizontal axis is the time. The system size is 128 sites of which 25 are filled with cars, the figure shows 8192 iterations. A small shot noise ($p = 0.005$) is applied and only one curb is present (arrow).

Let us now analyse the flow with one curb in the system. If there are curbs in the system ($r > 0$) then the originally existing translational symmetry is broken. Therefore in the case of one curb the jam occurs behind it (figure 4), and not at a random site as in the $r = 0$ case.

Having examined numerically the density region $\rho = 0.13 \ldots 0.5$ we have observed two different regimes. In the region $0.133 \leqslant \rho \leqslant 0.333$ two types of jams can exist: one with $\rho_{loc} = 0.333$ and $j = 0.66$ and the other with $\rho_{loc} = 0.5$ and $j = 0.5$. The former corresponds to the configuration

$\cdots$○●○○●○○●○$\cdots$

and the latter to

$\cdots$○●○●○●○●○$\cdots$

where ● denotes a car and ○ is an empty site. Both of them are stable modes. In the presence of random noise the system switches between these two modes (see figure 4), resulting in jams of different sizes. As we can see from figure 1 the flow rate linearly decreases in this region, this is related to the fact that the two jammed phases have weights linearly changing with ρ.

For $0.333 \leqslant \rho \leqslant 0.5$ only the lower density jam ($\rho = j = 0.5$) remains stable, since the global density exceeds the local density of the other jammed mode. As a result the flow has a constant $j = 0.5$ value in this density region (figure 1).

In the 'bulk' region (i.e., not in the jam) mainly the densest free moving mode ($\rho_{loc} = 0.1$ and $j = 0.5$) is present, which is produced when cars are released from a jam. This mode explains the plateau at 1/2 for a wide range of curb densities. The existence of two well-defined modes of propagation (jammed and bulk) is reminiscent of fluid–steam separation: here the role of the steam is played by the low-density state, and the jam represents the fluid phase. For low car densities there is a non-ordered car flow, which is the equivalent of the gas phase.

In conclusion, we have studied a one-dimensional traffic model with quenched noise (curbs). We showed that even a single curb causes jam 'crystallization' at sufficiently high car densities. We observed extreme sensitivity to the shot noise both at low and high curb densities.

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